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ABSTRACT

A methodology for busing to achieve school desegregation is described. Two different approaches are proposed: a student interracial contact score and a quota method. Travel time and number of children bused are proxies for busing costs. Useful data include travel time, school capacity, and student residences for each region and level of school. A higher ceiling on individual travel time allows greater balance within the area. For example, when contiguous districts are added, busing 25% of the students can achieve 95% desegregation with a 45-minute upper limit as opposed to 90% with a 35 minute upper limit. Portable classrooms are not very helpful nor is splitting schools into smaller grade spans. For the sample city, cost of raising desegregation from 41% to 85% was $16 million ($25 per student). The critical factor in reducing costs is the greater use of each bus by shorter trips and an efficient system of staggering school starting hours. Alternatives to daily busing are described. (DJ)

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PLANNING SCHOOL DESEGREGATION

Emmett Keeler

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A WORKING NOTE
prepared for the

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PREFACE

This is the final report on a study of school desegregation planning methodology, sponsored by the Office of the Assistant Secretary for Planning and Evaluation, Department of Health, Education and Welfare. Rather than recommending specific desegregation plans, we develop here a methodology to help school planners use their resources more efficiently, and to help governmental officials judge the scope and efficiency of proposed plans.

I am indebted to A. W. Bonner, J. H. Lindsey II, D. S. Pass, and A. H. Rosenthal for their technical assistance, and to V. Keeler and A. Pascal for helpful comments.

The data used to illustrate application of the methodology reflect one large urban area. We appreciate the cooperation of the staff of various public agencies in that metropolitan area, in providing information for this study.

SUMMARY

Many large metropolitan areas are faced with the problem of planning school desegregation. This paper concentrates on daily bussing as the means to achieving such desegregation. Even those opposed to bussing (and we briefly mention other alternatives) will agree that it should at least be done cheaply and efficiently. Our purpose is to develop a methodology that any area could use; the models and data-handling procedures were tested on an actual city. Following this summary is a checklist to help school planners.

In each city, at some point, a final decision must be made between desegregation and the financial and emotional costs of bussing. To help this decision, a wide spectrum of plans should be presented, each giving minimal cost bussing schedules for a different level of desegregation. In order to generate these plans, costs and ethnic balance must be quantified.

Thus, we first discuss how desegregation might be measured. We propose two different approaches: a student "interracial contact" score, and the more common "quota" method, which uses upper and lower limits on the proportion of minority students at each school. For bussing costs, we use as proxies the total travel time, and the number of children bussed.

The problem is formulated as a linear programming problem, using the quota method to gauge desegregation. To reduce the computations to manageable size, schools and children must be aggregated into regions. If the regions are large, there are few of them, and so the proper student flows between regions are easy to compute. However, large regions make travel-time estimates less reliable, and problems with segregation within the regions may appear, since the analysis assumes that each region is homogeneous and gives only the total number of students that go from one region to another. Which students in the region go, and to which school they are sent must be resolved for each region by hand after the choice of overall plan is made. (We show how this can be done on page 41.)

A large part of the effort was directed towards getting useful data. To make a plan, information is needed on travel times, school capacities and student residences for each region and level of school. We describe how we converted information from a variety of sources on the sample city into useful data, but we suggest instead that school systems collect their own information directly, being sure to keep it in a machine-processable form.

Sample plans were constructed with different sets of assumptions--whether contiguous districts are included in the area to be desegregated, whether new school building is allowed, what percent of students can be bussed, and what upper limit is set on individual travel times. Our object was to find out what effect each of these factors had on lowering total costs.

When the suburban contiguous school districts are included, not only does the overall percent majority increase, but greater desegregation is possible. The reasons are that more majority students are in range of the inner city, and fewer minority students need be bussed to majority regions within the central district. While the total number of students bussed is more than in the central-district-only plan, the percent bussed drops by about one third, for equal levels of desegregation. Other advantages of including contiguous districts are that school resegregation, through families moving to other regions, is less of a problem, and overcrowding is relieved at the same time.

Some bussing is already necessary due to overcrowding, and because some students live farther than walking distance from the schools they attend. To minimize the additional number of students bussed, the neighborhoods picked for bussing to another region should be as far from their own local schools as possible. Only five per cent of the central district's students are currently bussed, much less than the national average. To achieve 90% of possible desegregation, and at the same time relieve overcrowding, an additional 20% of the students need be bussed.

A higher ceiling on individual travel times allows greater balance within the area. For example, when contiguous districts are added, bussing 25% of the students can achieve 95% desegregation with a 45-minute upper limit as opposed to 90% with a 35-minute upper limit. (The 45 minute upper limit does not mean that all trips are that long--the average travel time in the case given is 20 minutes). Unless each school is required to have exactly the same proportions, there is little advantage in allowing rides longer than 45 minutes.

Curiously enough, new portable classrooms do not help much, either in reducing segregation or in cutting costs. For efficiency reasons, such new classrooms should be concentrated in transitional areas, rather than in the inner city or the suburbs. Another frequently mentioned alternative is grade reorganization--splitting schools into smaller grade spans. Unless most students can walk to several schools, this increases costs greatly; an enormous number of students must be bussed.

The ten year costs in this study average out to 20.6 thousand dollars per bus per year. For our sample city, the cost of raising desegregation from the present 41% to 85% was $16 million or about $25 per public school student. To further raise the index to 91% would cost an additional $4 million. The distance the busses travel has little effect on costs, compared to the total number of busses. Thus the critical factor in reducing costs is the greater use of each bus, by shorter trips and an efficient system of staggering school starting hours. The shorter trips allow each bus to make more trips, and to be fully loaded without student discomfort. The recommended method of staggering starting hours reduces costs to 40% of what they would be if all schools started at the same time.

We have assumed that the act of desegregation itself will not influence future choices of residency or school (public vs private). This is a crucial limitation, as in many communities, such family choices have continued to thwart meaningful desegregation. For this reason, we briefly discuss alternatives to daily bussing, and ways of reducing its resegregation impact.

A CHECKLIST FOR DESEGREGATION PLANNING

POLICY DECISIONS:

1. Define the groups to be desegregated.

2. What level of desegregation is required? desired? How is desegregation measured? (p. 3).

3. Are plans mandatory (through daily bussing or special subject schools), or are they voluntary (through magnet schools and desegregation incentives)? (p. 34)

4. What are the constraints on bussing? Are they costs, individual trip times, percentage bussed, one-way or two-way flows?

5. How much money goes into desegregation planning? Will outside consultants or software be used? Do you have machine records of school data, and access to a computer?

6. Decide whether to try:

   Grade reorganization (raises costs greatly, p. 31).

   New construction (portable classrooms have little effect on costs or numbers bussed, p. 29).

   Certain schools allowed to differ from limits (except for obviously isolated schools, doesn't reduce costs much, and may add to resegregation problems). Enlarging district by including contiguous school districts (the added majority districts have a significant effect on desegregation, and resegregation problems are reduced, p. 22, 27).

7. Will teachers be trained? What will happen to special programs for the poor or minorities?

MAKING THE PLAN

1. Decide on the regions for analysis: schools, neighborhoods, or geographical areas? (If the regions are too small, the problem is unwieldy, because of the large number of regions, p. 7.)

2. Find data by region on:

How many students of each group are resident. This can be done via the census or school records (we recommend making your own records, p. 14).

School capacities (must decide how much overcrowding is permitted).

Travel times between regions (p. 10).

3. Find data on costs: Drivers salaries, purchase or lease cost of busses, land, maintenance.

4. Decide on flow between regions: The simplest way is to pair regions by hand (with one region allowed to be paired with several others). For more efficient results, we used a linear programming technique (p. 37).

5. Schedule busses to carry the flows.

Staggering school starting hours cuts costs greatly (p. 53).

Decide how the individuals in the region will be selected. (We recommend that small neighborhoods rather than individuals be selected. By proper choice, bussing within regions, and segregation within regions can be minimized.)

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INTRODUCTION

Today many large metropolitan areas are faced with the problem of planning school desegregation. To help them with this problem, we here discuss some techniques for finding efficient schemes for scheduling pupil-to-school assignments so that the schools of an area may be less segregated than they would be if children were to go to their current schools. The methodology that we present was developed and tested by constructing a variety of sample plans for an actual city.

Segregation is defined to be a function of the relative proportions of "minority"* and "majority" students attending a given school. "Minority" denotes here black, Indian, or Spanish surnamed students as these characteristics are determined by the U.S. Census Bureau. "Majority" includes all other students.

We discuss two distinct situations: (1) desegregating the schools in the largest school district in the area, in most cases the Central District, referred to as CEND; (2) desegregating the schools in both CEND and in all Contiguous Districts (CEND + CD). In the latter situation, it is assumed that students living in one district may be assigned to schools in another. This procedure will

*Minority is in quotation marks because in many metropolitan areas they are or will become the majority of the school population.

generally lead to greater balance for the area, but will present greater administrative and political difficulties.

One should not insist that each school in the area have exactly the same ethnic ratio. Society has many goals besides desegregation, such as provision of high-quality education, using money and students' time efficiently, compliance with community preferences, and the like. The sacrifices that desegregation requires in terms of all these other goals will here be called "bussing costs."

One consequence which significantly affects the long run viability of plans is resegregation through residential moves and private schools. In a later section on re-segregation, we discuss alternatives to daily bussing of pupils outside their neighborhoods that may have more widespread appeal. However, the body of the work deals only with desegregation as achieved by such bussing.

FORMULATION OF THE PROBLEM

The final decision of level of desegregation, then, involves the tradeoff between desegregation and bussing costs. To make this decision easier, a wide spectrum of efficient plans should be presented, each giving minimal cost bussing schedules for different levels of desegregation. To permit these plans to be generated systematically, the factors to be traded off must be quantified.

Measures of Segregation

A measure of segregation must be chosen which reflects what we want to achieve: one set of enrollment proportions in the region will be said to be more balanced if we prefer it, other things being equal. There will be no disagreement as to what are the extremes of segregation--an area is desegregated if every school has the same ethnic ratio, and segregated if every school is entirely minority or entirely majority. However, intermediate judgments are less clearly defined. For example, in a region with three schools that have "minority" of 45 percent, 45 percent, and 60 percent more segregated than a region with three schools that have "minorities" of 42 percent, 50 percent, and 58 percent?

One crude index which has been widely used is the percentage of "minority" students in the area going to predominantly "majority" schools. The problem with this index is that it is very dependent on the percentage of minority students in the region. For example, in a predominantly black town like Gary, Indiana, the most balanced allocation of students to schools would have no blacks attending white majority schools.

A more valid approach is to set a segregation score for each student and let the desegregation index (DI) in the area be the sum of the individual student scores. For example, let the segregation score, $e_i(p)$ be the proportion p of majority students in the school that minority student i attends.

In a school with N students, with m of these minority students, $e_i(p)$ will be $\frac{N-m}{N}$ for each minority student. Thus, the sum for the school will be $\frac{(N-m)(m)}{N}$. This "score" or number will range between 0 and .25N and makes it possible to compute a desegregation index for the area. Since this function is concave, the sum for the area (DI) will be maximized when all schools have the same ethnic balance.*

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*In [1], Cisin gives the index a statistical justification. Let n_i represent the total number of students at the ith school, and N the total for the district. Let p_i represent the percent majority at the ith school, and P the percent majority for the district. Then the expected number of majority students at the ith school is n_iP and hence the χ^2 is $\Sigma \frac{(n_iP - n_iP_i)^2}{n_iP}$, which is $N - \frac{DI}{P(1-P)}$.

Different school districts can be compared to determine which gives stronger evidence of being segregated by looking up the χ^2 in a table, where the number of degrees of

To compare different school districts, the DI should be normalized by dividing by its maximum value NP(1-P). The normalized index is 0 if all schools are entirely minority or majority, and is 1 if all schools have the same proportion of majority students. In making such comparisons, we must look carefully at the larger area involved. For example, most city school districts could greatly increase their DI by dividing up into gerry-mandered smaller districts that are largely minority or majority. This should not be encouraged. One way to gauge the desegregation value of district expansion is to compute the DI for areas which include various sets of contiguous school districts, as we have attempted in this paper.

Another approach to desegregation is by limit proportions, sometimes called quotas, which are upper and lower limits on the proportion of minorities in each

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freedom is 1 less than the number of schools in the district.

For most purposes, however, the index divided by its maximum value NP(1-P) is a better measure than the χ^2, since we consider a district with 2 schools of size 1000 and certain majority proportions as segregated as a district with 2 schools of size 2000 and the same proportions. The imbalance in the latter district is less likely to have been caused by chance fluctuations and so its χ^2 will be larger.

Cisin is mistaken in suggesting that the F-test can be used, since the χ^2 distributions he places in the numerator and denominator are dependent. Also, the maximum value NP(1-P) should not be called the total variance and the index is not the variance due to segregation.

school in the area. This is the approach used in legal directives for desegregation. It is easily understood, and permits us to use simple mathematical techniques to solve the minimal cost bussing problem. If the limits are the same for each school in the area, then the narrower the allowable range, the less segregated the region. If limits are different for different schools, comparisons between desegregation plans must be made by computing the district-wide DI, as discussed above.

Bussing Costs

There are two distinct types of bussing costs. The first type includes the financial costs of buying, maintaining, and driving the busses. We show in Appendix B how careful scheduling and staggering school times allow busses to be used for several trips, morning and afternoon. Such measures may reduce costs, but costs will remain proportional to the number of children bussed. An interesting fact is that cost is less dependent on the distance traveled than on the number of busses since much of the cost is fixed capital, and drivers' salaries do not depend heavily on distance traveled.

The second type of costs includes the dislocation costs for the children and their families--the inconvenience and time in waiting for and riding the bus (which is generally entirely minority or majority), the emotional costs for children in leaving their neighborhood, and so forth. It seems unfair and unwise to subject children to too long a ride, and several states have set upper limits on the amount of time allowed for trips. In this study we will use the criterion that no children will ride more than 45 minutes (one way) more than 5 percent of the time.*

Thought and care can reduce these dislocation costs. Bussing should be done by families and neighborhoods, rather than by individual children. Time on the bus might be made more valuable by utilizing teachers' aides or installed equipment. In this study, we will use the number of children bussed as a proxy for financial costs, and the total on bus time of all children as a proxy for the dislocation costs.

Aggregation

Because of the vast number of schools and children in a large city, it is too expensive to get the computer to assign each individual child to a school. Schools and children must be aggregated into neighborhoods or regions. The mathematical programs can then compute the optimal flow of children from each region to schools in other regions. Once a plan is selected, the details can be filled in. We can then determine which children (or subneighborhoods) stay in the region and which leave, which schools in the assigned region will provide transportation, bus pickup points, and so forth. Such decisions

*Because of varying traffic conditions, it is impossible to insist that rides be less than a fixed limit all the time.

might be made locally and will be based in part on factors not considered in the larger analysis.

For simplicity the same regional partition of the district was used for both schools and children. The size of the regions depends on two competing considerations. The larger the regions are, the fewer there will be, and the easier the mathematical problem of choosing the optimal flow.* However, if the regions are too large, there are more problems in estimating travel times and in internal regional assignments.

In the central school district analysis, we chose 44 regions and attempted to make them small enough so that travel time estimates from the center of one to the center of the other could be used as approximate travel times from anywhere in one region to anywhere in the other. We tried to create districts that were roughly square, were equal in size, and contained approximately the same number of schools. We could not satisfy all these criteria simultaneously, and some of the more sparsely inhabited regions--are much larger than the others. In these regions internal bussing will be required, but because the number of students involved is so small and normal bussing probably required anyway, we have ignored this.

* The programs use the flow between regions as variables, so the size of the computation problem varies directly with the square of the number of regions.

A Variety of Plans

We must generate a great number of plans to facilitate decisions regarding (1) the size of the area to be integrated (CEND or CEND + CD), (2) whether different rules and guidelines should apply to different levels of schools, (3) whether new school construction should be ordered, and (4) the amount of desegregation required. When the real costs and effects of various alternatives are known, decisions can be made more rationally.

There are two approaches to efficiency. We can set a limit to bussing and minimize the segregation possible within that limit, or we can fix a certain level of desegregation and find assignment plans that achieve that level at minimum bussing cost.* For mathematical convenience, we have chosen the latter approach. For each target level of desegregation, we will compute two solutions corresponding to the two types of bussing costs: one will minimize the number of children bussed, and the other will minimize the total travel time. A third solution will permit money to be spent on new school and classroom construction and minimize total estimated financial costs.**

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*The results of the two approaches can be made the same, through successively better estimates of the limits which result in the desired cost.

**Based on estimated costs for portable classrooms and for bussing derived from data given by the test school district.

PROBLEMS WITH DATA

In the mathematical formulation of the problem,* travel times, school capacities and numbers of children of every level and type in each region are assumed known. However, finding this information became a major part of the study effort. Much information on these subjects has been gathered by various people and agencies, but it is often difficult to discover, get permission to use, and put into machine-usable form. We shall describe how we estimated the numbers for the test area, and suggest some other ways that planners inside school systems might use.

Travel Times

Travel time estimates were made from points near the center of each region to points near the center of each of the other regions, using a minimum time path program developed for the test metropolitan area. This program used a computer model of traffic developed by that metropolitan area's State Division of Highways, with specific metropolitan data taken from a citizen questionnaire filled out at the time. The model had 1200 points located throughout the metropolitan area, from which we have chosen the 65 points nearest to the center of the CEND regions and to the centers of the adjoining school districts. There are two sets of travel times, representing peak and off-peak traffic conditions. This data was verified by checking it against

*The problem is formulated in Appendix A.

the continuing surveys of travel times on city streets by the City Traffic Department. There was good agreement on the times checked.

The survey of travel times also contained information on variability of trip times. In both peak and off-peak hours, the 95 percent confidence interval for trip times has the mean ± 8 percent on the average. Freeways can be expected to be somewhat more variable, so that we have taken mean travel times ± 10 percent as the number to be compared with the 45-minute upper bound on travel times. Thus, trips can be expected to last longer than this no more than 1 time in 20. Loading and unloading the busses, which may involve several pickup points and several destination schools, has been assumed to take five minutes. Errors by using center-to-center travel times as a proxy for pickup-to-school times should not be more than five minutes, if there are a reasonable number of schools in the district to which the bussed students can be assigned. This will certainly be the case for elementary students unless grade reorganization takes the form of one-grade schools.

In Fig. 1, we demonstrate the assignment method on one of the more difficult cases that occurred. When the contiguous districts were also considered, the central school district was split into only twenty regions. These larger regions averaged five miles on a side. One schedule assigned a third of the elementary school majority students in region 11 to schools in region 4. The problem is that

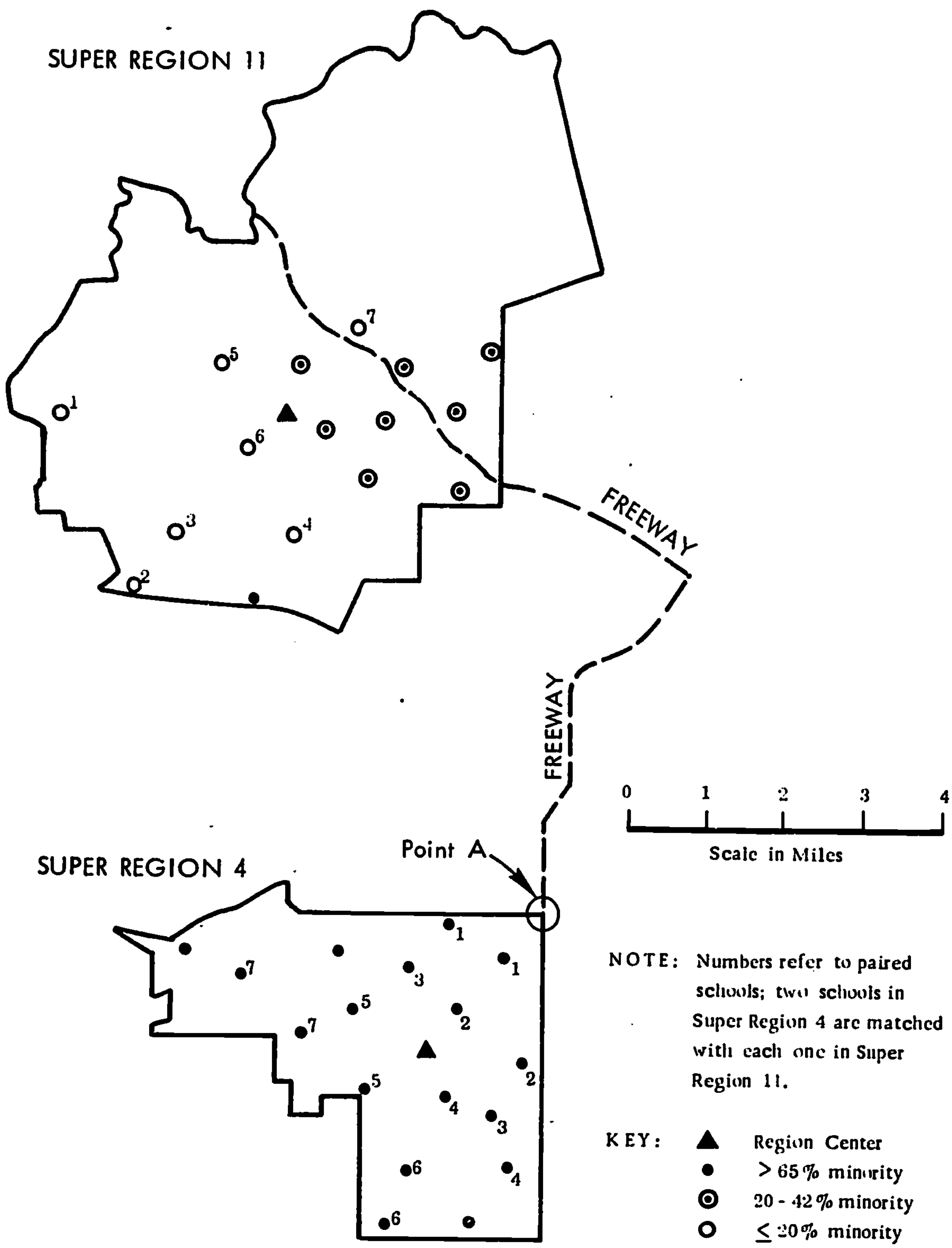

Fig. 1--Distribution of Elementary Schools in Super Regions 4 and 11

the majority students are bunched away from the freeway in region 11, and the shortest travel times are achieved by going to the freeway, and using it as much as possible. However, we pair the highly majority schools in region 11 with target schools in region 4 (all of whose schools are over 90% minority).* The table below shows that the largest excess over center-to-center times is three minutes.

Differences Between School and Region Center Travel Times

School Number	Minutes that School Travel Time to A Exceeds Center Travel Time to A	Minutes that School Travel Time from A Exceeds Center Travel Time	Difference Between School-to-School and Center-to-Center Travel Times
1	6	-4	2
2	5	-3	2
3	4	-1	3
4	0	0	0
5	-1	2	1
6	0	3	3
7	-4	4	0

For the upper time limit to be violated, all of the following occur: the center-to-center times must be close to the maximum, the regions must be large, the destination schools must be located at the end of the region farthest from the origin, and the children being bussed must live in a "pocket" in the origin region farthest from the destination. The regions in the central district were fairly homogeneous, and even if they were not, the students in the "pocket" will generally be used to desegregate

*The example shows how desegregation within regions can be aided by selecting the residential districts from which bussed students come, and the schools they go to. The plans the schools in their own region, and hence are less likely to be used. The combination of circumstances is most unlikely.

To sum up, we use a mean center-to-center time of 30 minutes as the upper limit on bus times. With five minutes allowed for traffic variability, five minutes for loading and unloading, and five minutes for variations from center-to-center times, we can expect almost all trips to be less than 45 minutes most of the time. Alternate limits of 25 to 35 minutes average center-to-center times are also investigated.

Student Residences

The census and school records are the two main sources of information on student residences. The main theoretical problem with the census is that it is valid only at ten year intervals. In addition, a number of estimations are needed to convert census data, which is broken down by age and census ethnic groupings, into data broken down by public school grade and different ethnic groupings. Moreover, some information--the numbers of children with Spanish surnames or Oriental children and the breakdowns to one-year age groups--do not come until later counts and are not yet available for 1970.

Schools have records on where each student lives, but it is too expensive to get this into (machine) usable form, i.e., summed over our regions.* If each child went to a

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do not consider segregation within regions, but a detailed look at those regions thought to be most segregated showed that segregation within regions could be eliminated with very little additional bussing.

*With infinite money, we could address-census block match each student and sum over our regions.

school close to home, one could use the school figures collected for HEW, under Title VI of the Civil Rights Act of 1964, as proxies for neighborhood figures. Unfortunately for this procedure, many children now ride busses to school or walk long distances. As bussing becomes more widespread, schools will be even poorer proxies for student residences. However, schools appear to be the best potential long-run source of student residence information. We give some recommendations below on how school systems contemplating desegregation can gather the information they will need.

In this study, we examine both the census and HEW records to discover the problems in getting their information into workable form. While the records are not strictly comparable--the school data was from the fall and the census from the spring of 1970--they were fairly close. Following HEW requests, we used the census data in generating the bussing plans.

Next we discuss the practical problems involved in obtaining the numbers of minority and majority students living in each of our "regions" for each level of school.

The census: We need data on where students live broken down by grade and by our own ethnic groupings. Unfortunately, the first count of the 1970 census only gives data by age, sex, and "all persons" or "black." Thus we are faced with the estimation problems presented in Fig. 2.

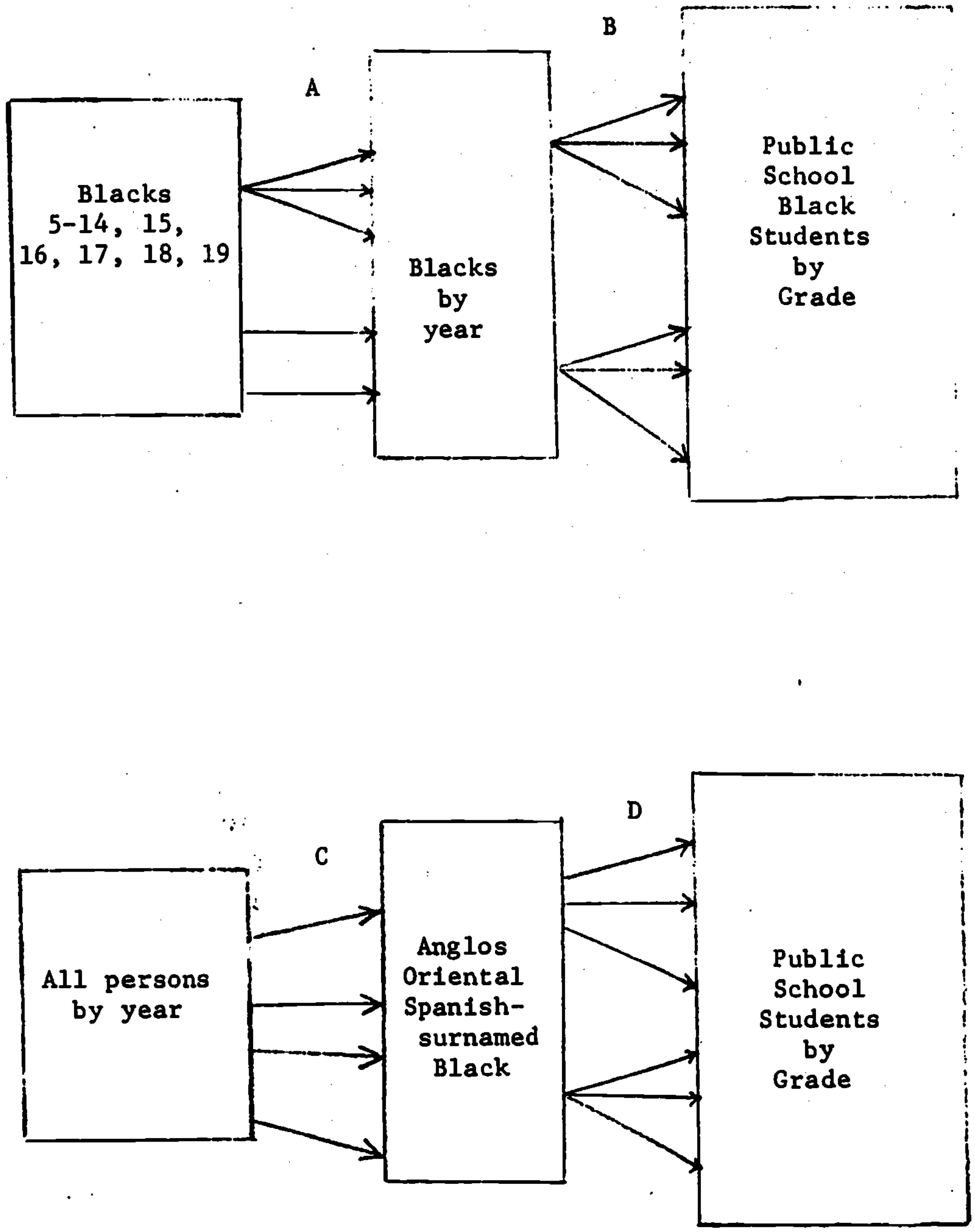

Fig. 2 -- Steps in Converting Census Data

The data on blacks in the age levels we consider (5-19) comes by sex and age groups of 5-14, 15, 16, 17, 18, 19. The easiest way to distribute the 5-14 span into individual years is to use the past black birth statistics in Vital and Health Statistics, slightly adjusted for deaths. The finer 1960 census age-distributions are not so useful for this purpose, since the height of the "baby boom" has now shifted from 7 to 17. However, we do use the state 1960 nonwhite age-to-grade ratios (Census, Table 101) to place the black students into public school grades. This approach assumes that the major determinant of age-to-grade distributions--promotion policies--has not changed much in the last 10 years. Since there were significant differences in the age-to-grade patterns for black boys and girls, the two sexes were sorted separately and added.

A problem in dividing "all persons" into minority and majority is that earlier censuses used a quite different division, white versus nonwhite, and the more detailed ethnic information has not yet come out of the 1970 census. After subtracting the blacks, we estimate the Orientals and Spanish-surnamed in each region from their percentage in the school records of that region. Placing the majority students into public school by grade is also done through the age-to-public-school-grade ratios derived from the state census, Table 101.

It proved to be fairly complicated to get the information from the 1970 First Count Summary Tape summed over our own geographic regions. We first selected the county from the state tape, and sorted it by census tract, block group, and enumeration district. We drew up a table of census tracts that fell into each of our 65 regions, and used a prepared program to recode the sorted Tape, placing the region number where the township code normally would be. We then sorted by region, and used another prepared program, slightly modified, to sum up the "all persons" and "blacks" by age over the regions. A slight distortion was caused by the fact that some census tracts cut across different school districts, and hence different regions. In this case we placed the tract in the region where most of its inhabitants lived.

School Records. The school records are easier to use, as they give their data in terms of grade and five distinct ethnic groups (white, black, Oriental, Indian, Spanish-surnamed). The only difficulty was placing the schools into census tracts. This we did by address-matching, using a prepared program. We could then use our table of tracts into regions and add up the students of each type. If schools could keep records on which region students lived in, this task would be very much easier.*

*All that is required is a standard map, with individual teachers supplying information on how many students come from each area. They would only have to know whether the child walked or on which bus he came.

School Capacities and Overcrowding

We were not able to get information on school capacities directly. As a proxy for capacity, we used the school enrollments. However, the number of students living in a region were derived from the census and did not balance with the enrollments of the schools in the region. This was particularly true of high school--several of the geographical regions had no high schools.

The data on capacities were therefore adjusted by assuming students in overcrowded regions would walk to high schools in adjacent regions within 1 1/2 miles of their homes. Planners inside school systems should be able to get better information on school capacities, as such information must already be in use.

RESULTS

The computer runs show the tradeoffs between amount of desegregation and four important parameters: limits on individual travel times, limits on percent of students bussed, whether the area to be desegregated includes the contiguous districts, and whether new building is permitted. We will discuss each tradeoff in turn.

Maximum Desegregation

For Table 1, desegregation, as measured by the DI, is maximized subject only to the upper limit on individual trip times. The solutions cause a great number of children to spend long times on the bus, but are interesting since they set limits on what any desegregation plan can achieve.

The column, "Time Limit," gives the maximum allowable single trip travel time. As explained above, this is 15 minutes more than the longest trip would last, on the average, when loading and unloading times are excluded. The overall area percentage is given in "percent majority in area." Desegregation would be total if every region had that majority percentage. After the DI has been maximized, the regions of the area fall into two large groups, with each region in a group having identical percent majority. For example, in the first line, the 71.4 percent of the students in the central city would go to 40.3 percent majority schools, the 27.3 percent of the students in the farthest suburbs would attend 82.4 percent majority schools,

Table 1

MAXIMUM DESEGREGATION SUBJECT TO TRAVEL TIME CONSTRAINTS
(Peak Travel Times)

	Time Limit (Minutes)	Percent Majority in Area	Range of Percent Majority: Low	Range of Percent Majority: High	Percent of Students in Schools at Range Limits: Low	Percent of Students in Schools at Range Limits: High	Percent of Students Moved	Desegregation Index: Fall 1969	Desegregation Index: Final
Central District (44 Regions)									
Elementary	35	52.2	40.3	82.4	71.4	27.3	37.4	.39	.856
	40	"	42.7	77.4	67.8	25.3	40.0	"	.910
	45	"	45.9	69.4	71.4	25.3	44.3	"	.958
	50	"	50.7	55.3	68.5	31.5	49.1	"	.998
Junior HS	40	55.3	45.0	82.4	69.6	26.9	37.6	.455	.890
	45	"	47.8	75.6	73.1	26.9	43.1	"	.938
	50	"	52.3	61.9	68.7	31.3	48.0	"	.992
Senior HS	40	58.3	48.4	84.3	70.3	25.9	40.0	.442	.900
	45	"	51.4	77.9	74.1	25.9	42.4	"	.945
	50	"	56.3	62.7	69.5	30.5	47.6	"	.997
Central District plus Contiguous Districts (41 Regions)									
Elementary	35	59.7	54.6	87.0	72.3	5.2	39.1-	.432	.963
	40	"	57.0	70.8	80.6	19.4	39.1+	"	.988
	45	"	59.6	60.0	0	100.0	39.8	"	1.00
Junior HS	35	62.3	56.5	88.2	73.1	5.5	37.8	.466	.952
	40	"	59.3	76.5	79.8	20.2	38.5	"	.980
Senior HS	35	65.2	58.8	90.4	72.7	5.1	36.7	.478	.950
	40	"	61.8	78.8	80.5	19.5	37.9	"	.980

and the remaining 1.3 percent would attend schools with percent majority between 40.3 and 82.4.

These plans are absolutely optimal in the sense that every minority student within range of the 82.4 percent majority schools is bussed to them, and every majority student within range of the 40.3 percent majority schools is bussed there. As the maximum allowable travel time increases, the range of percent majority narrows, segregation decreases and the number of students moved increases.

The results look better when the contiguous districts are included in the area to be desegregated. The main reason for this is that several largely majority contiguous districts are located close to the areas of heaviest minority concentration in the central district. In addition the overall percent majority is higher, so that the largely majority regions within the central district don't have to change as much. Finally, the model is biased in favor of the contiguous districts area, because the regions in that case are much larger and internal region segregation is not considered. As discussed below, an additional three percent of total students would have to be bussed to relieve internal region overcrowding and segregation.

Lower Levels of Bussing: No New Construction

The bounds of percent majority in Table 3 is preselected by slightly relaxing the tightest possible limits given in Table 1. We shall use the first column of Table 3 as an example to explain the entries in the tables. The upper half of this column summarizes the results on minimized total student travel time with narrow limits on percent majority, and new construction not considered.* In this case, the average travel time (excluding loading and unloading) is 20.0 minutes. The percent of all students bussed is 40.6 while 50.8% of minority students are bussed. The reason for this difference is overcrowding of the predominantly minority inner city schools. Thus, many minority students are bussed out of these schools, and a smaller number of majority students are bussed in. The desegregation index is .909, which means that the average minority student is in a school which is 50% majority (55.3% x .909). The second column differs from the first only in that the number of students bussed, and not the total travel time is minimized.

When the contiguous districts are included in the area to be desegregated, much narrower limits on school percentages are possible. Just as in Table 1, the percent

*Because of the assumption that students should not ride three-abrest on long trips, the minimum travel time solution generally requires fewer busses.

Table 2

MINIMAL TOTAL TRAVEL TIME AND NUMBER OF BUSSED STUDENTS

ELEMENTARY SCHOOLS

Limits of Percent Majority	Central District (44 Regions) (52.2 Percent Majority in Area)				Central District plus Contiguous Districts (41 Regions) (59.7 Percent Majority in Area)			
	Narrow (42-75)		Wide (35-80)		Narrow (55-65)		Wide (51-75)	
Minimize:	Travel Time	Students Bussed	Travel Time	Students Bussed	Travel Time	Students Bussed	Travel Time	Students Bussed
No New Construction								
Travel Time per Student (Min)	20.5	23.0	13.5	22.1	22.4	24.3	20.3	20.9
Percent Bussed	35.1	32.5	26.5	25.7	33.1	32.2	26.7	26.2
Percent of Minority Bussed	33.7	30.2	24.3	23.1	38.8	37.7	30.7	29.7
Ethnic Balance	.9086	.9067	.8154	.8472	.9903	.9906	.9482	.9485
New Construction Allowed								
Travel Time per Student (Min)	20.8	23.9	17.9	22.6	22.4	24.9	19.6	21.7
Percent Bussed	36.0	20.9	23.7	21.4	30.9	30.6	24.7	23.6
Percent of Minority Bussed	n.a.	30.8	n.a.	24.4	44.0	39.5	34.0	35.8
Ethnic Balance	n.a.	.9066	n.a.	.8310	.9901	.9899	.9481	.9501
Construction (Student Spaces)	7,756	14,402	15,292	22,133	18,179	11,081	17,733	22,870
Total Students	362,155				538,439			

Table 3

MINIMAL TOTAL TRAVEL TIME AND NUMBER OF BUSSED STUDENTS

Junior High Schools

Limits of Percent Majority	Central District (44 Regions) (55.3 Percent Majority in Area)				Central District plus Contiguous Districts (41 Regions) (62.8 Percent Majority in Area)			
	Narrow (45-78)		Wide (40-83)		Narrow (58-68)		Wide (54-78)	
Minimize:	Travel Time	Students Bussed	Travel Time	Students Bussed	Travel Time	Students Bussed	Travel Time	Students Bussed
No New Construction								
Travel Time per Student (Min)	20.0	22.1	17.7	22.7	22.3	24.2	19.8	20.3
Percent Bussed	40.6	37.5	34.9	31.8	33.3	32.9	26.3	25.9
Percent of Minority Bussed	50.8	44.0	40.7	37.5	50.9	50.2	40.6	39.8
Ethnic Balance	.9090	.9104	.8332	.8560	.9895	.9899	.9444	.9459
New Construction Allowed								
Travel Time per Student (Min)	19.8	25.0	n.a.	25.3	22.3	25.0	19.8	21.3
Percent Bussed	39.8	32.1	n.a.	26.5	32.0	30.9	24.8	23.8
Percent Minority Bussed	47.5	37.6	n.a.	n.a.	51.0	49.8	41.3	39.5
Ethnic Balance	.9057	.9069	n.a.	n.a.	.9895	.9897	.9447	.9461
Construction (Student Spaces)	2,099	11,985	n.a.	9,112	4,891	7,126	5,400	5,744

Table 4

MINIMAL TOTAL TRAVEL TIME AND NUMBER OF BUSSED STUDENTS

SENIOR HIGH SCHOOLS

Limits of Percent Majority	Central District (44 Regions) (58.3 Percent Majority in Area) Narrow (48-80)		Wide (43-85)		Central District plus Contiguous Districts (41 Regions) (63.2 Percent Majority in Area) Narrow (60-70)		Wide (56-80)	
Minimize:	Travel Time	Students Bussed	Travel Time	Students Bussed	Travel Time	Students Bussed	Travel Time	Students Bussed
No New Construction								
Travel Time per Student (Min)	18.1	21.6	16.2	22.6	22.6	25.1	19.1	20.8
Percent Bussed	42.7	38.9	37.4	34.2	32.0	30.7	25.9	25.1
Percent of Minority Bussed	50.5	46.8	42.1	41.1	54.2	51.9	46.6	44.8
Ethnic Balance Index	.9103	.9134	.8367	.8627	.9895	.9895	.9452	.9458
New Construction Allowed								
Travel Time per Student (Min)	18.4	26.0	16.1	24.4	21.9	25.7	19.1	20.0
Percent Bussed	41.8	31.0	34.7	27.0	31.2	30.6	24.8	23.8
Percent of Minority Bussed	50.4	42.4	41.7	38.2	57.7	51.9	46.8	46.2
Ethnic Balance	.9092	.9126	.8353	.8591	.9898	.9895	.9451	.9472
Construction (Student Spaces)	379	13,023	4,415	10,943	6,195	0	2,393	3,824
Total Students	125,368				191,315			

bussed in the table is slightly lower than it would be if the additional bussing required to relieve internal region segregation were included.

Figure 3 illustrates the tradeoff between achieved desegregation and percent of students bussed. Each point represents a case in which the total number of students bussed is minimized subject to a 45-minute upper limit on travel times and new construction is not considered. The lowest point on each line represents the status quo, and the highest point represents the maximum possible desegregation within the 45-minute upper limit. The intermediate points represent cases with desegregation limits less than the best achievable. The vertical scale represents the percent majority in the school the average minority student attends. This emphasizes the advantages of including the contiguous districts--both the DI and the percent of majority students are higher in the larger area.

Further work is needed to determine the shape of the real curve to the left of the point where the polygonal curves level out. Can we reduce the percent bussed without lowering the percent contact very much, or are the attainable states really as pictured in Fig. 3?*

*Because of the form of the problem, the real curves of solution must be convex, but that is not much of a restriction. E.g., the percent of elementary students bussed in the larger area to achieve 50% contact could be from 9% to 21%.

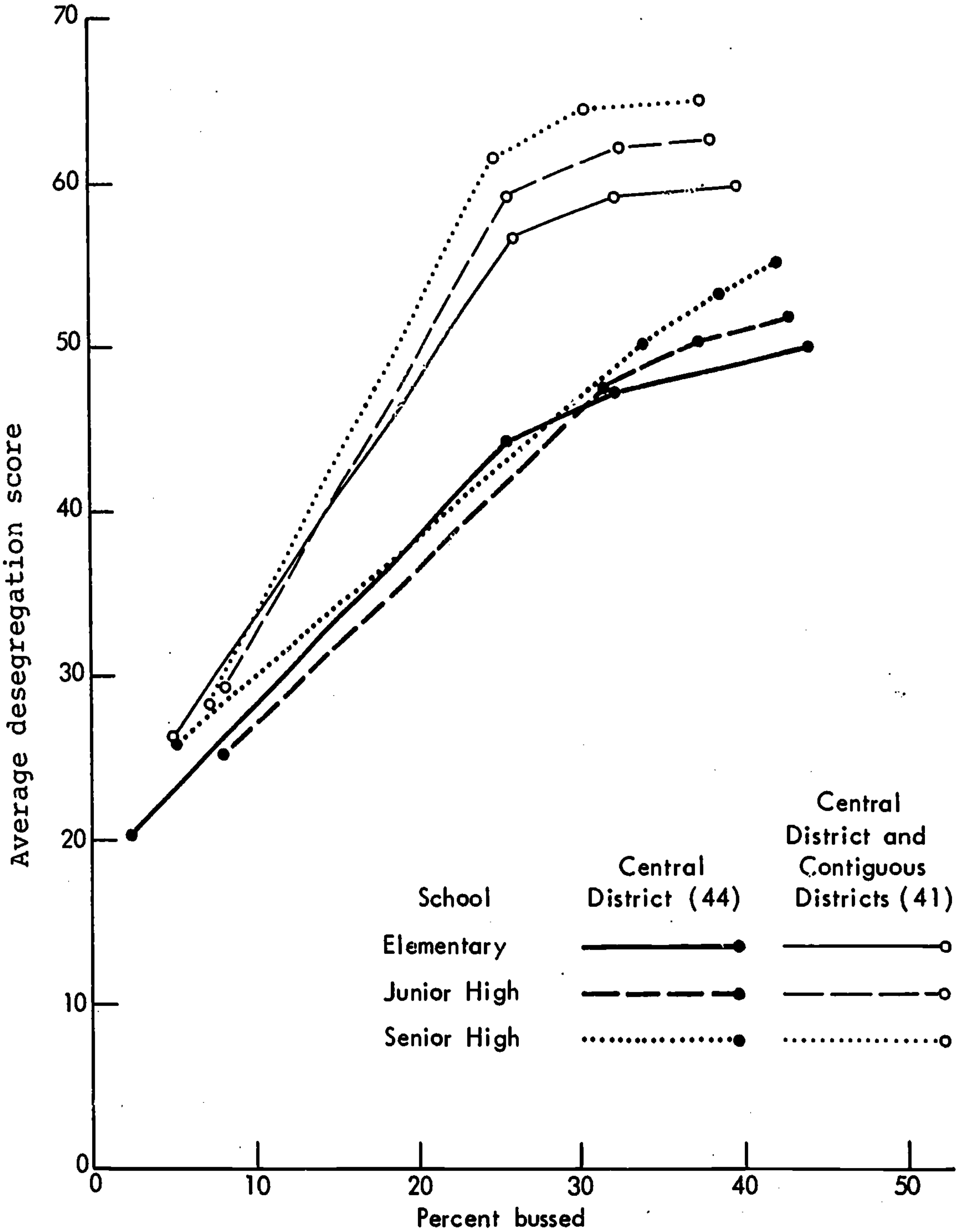

Fig. 3--Desegregation Achieved as a Function of Bussing

For each case listed in the tables, the computer gives more complete results than we have listed. For example, it tells how many students go from each region to each other region, which region-by-region departures from the uniform percent majority limits would reduce bussing the most, and much more.

New Construction

The only additions to classroom space considered were portable classrooms, located in current school grounds. For the purposes of minimizing the combined cost of bussing and classrooms, we assumed that bussing a student costs twice as much as placing him in a portable classroom.*

Continuing down the first column of Table 3, we see that travel time per student and percent bussed fall slightly and that portable classrooms are used by 2,099 students, about 1.5 percent of the junior high school students. By looking at the other cases, we see that there is surprisingly little building scheduled. There is a 1% to 7% reduction in the percent bussed, but the total travel time and combined costs do not change much.

*A two-story portable classroom for 30 students costs $23,000. Making the conservative assumption that additional maintenance adds $10,000 over a ten-year period, the total ten-year cost is $110/student/year. (No costs are given for reduced playground space.) Cost analysis has shown that a bus, which will carry about 215 students/ day has a ten-year cost of $26,000 per year. Assuming that a student's time is worth what the city spends to

In the analysis, the costs of new construction are understated. Why then does it have so little effect? No new construction was absolutely necessary, since the desegregation targets set were achievable without it. Local irregularities are ignored by the program, and "pocket" imbalances could not be relieved by building new schools between pockets, since construction was scheduled only at old schools. It wasn't valuable to make half as many students ride twice as far by building halfway along long trips, since even the travel times would increase because of loading and unloading.

Other Limits on Individual Travel Time

Runs were made with upper limits on student trips of 35 and of 60 minutes, instead of 45 minutes, to test how much would be lost orggained by varying the restriction. With the shorter time limits, maximum possible desegregation is reduced.* By extrapolating the results, it can be shown that levels of bussing achieving DI's of .95, .93, and .92 with a 45-minute upper limit, achieve only .93, .87, and .82 with a 35-minute upper limit.** While trips longer

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educate him (about $1.00/hour), the cost of his time on the bus is about $90/year. Thus the total bussing cost is about $215/student/year. These calculations are rough, but fortunately the results are not sensitive to the ratio of classroom/bus costs.

*This is the tradeoff given in Table 2.

**The scores are elementary, junior and senior high, respectively.

While trips longer than 45 minutes are necessary if total desegregation is required, at DI levels around .9, the 1-hour upper limit trips have only a slight advantage over the 45-minute trips. Thus, to achieve DI's at these levels efficiently, the upper limit of 45 minutes is about right.

Grade Reorganization

It is sometimes alleged that smaller groupings of grades, down to even single-grade schools, make school desegregation work better. However, if the same grades are to be desegregated in the "reorganized" school system as in a conventional one,* such reorganization will only increase the number of busses required, student travel times and number of students bussed. Consider a mild form of reorganization, splitting K-6 into K-3 and 4-6. Table 5 presents an example for a two-region district. We assume that each child can walk to the school in his own region, but not to the school in the other region. The price paid for reorganization is that 50 percent of the students must be bussed, since the desired final result is schools completely segregated by age. For desegregation alone, without reorganization, only 17 percent of the students are bussed.

*Some of these plans designate K-3 as neighborhood schools, not to be desegregated. Perhaps this is why people favor them. However, if one does not want young children bussed, a better plan is to desegregate only grades 4-6 at the K-6 elementary schools.

Table 5

ADDITIONAL BUSSING REQUIRED BY GRADE REORGANIZATION

Type and Grade of Student	Student Residences Region 1	Region 2	Flow Necessary Without Grade Reorganization 1 → 2	2 → 1	Flow if School 1 is K-3 School 2 is 4-6 1 → 2	2 → 1
Minority K-3	100	200	0	50	0	200
Mincrity 4-6	100	200	0	50	100	0
Majority K-3	200	100	50	0	0	100
Majority 4-6	200	100	50	0	200	0
Totals	600	600	100	100	300	300

students are bussed.

With smaller grade spans, the disadvantages increase. With one-grade elementary schools, 5/6 of the students must be bussed. Loading and unloading times would increase since busses must go over larger areas to pick up students of only one grade, or, if they pick up students of several grades, they will have to deposit them in several schools. Flexibility in deciding who is to be bussed and where is decreased. The distribution of travel times would be more spread out, since schools and home areas could no longer be matched up as in Fig. 1. One possible advantage is equity--since most people are bussed, the dislocation costs are more evenly distirbuted.

Financial Costs: Bus Scheduling

It is the number of busses, not the distance they travel, that is important in determining costs.* Thus, efficient schedules, which permit busses to make many trips each day, can cut costs greatly. At present, there is no feasible way of determining the optimal schedule for a given set of student flows. What follows are the main recommendations from Appendix B, which discusses the art of generating "good" schedules.

If students go to school in shifts, the number of busses can be reduced to slightly more than the number required by the largest shift. Thus, school starting hours should be staggered so that each shift has roughly the same number of students. The runs minimizing total travel time are cheaper to implement since the shorter times allow the busses to make more trips, and to carry more than 60 students more often. (The busses can hold

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*Drivers are already paid for a full day of work. As shown in Appendix C, the costs of buying busses, insuring and parking them, is much greater than operating and maintenance costs. For example, using 10-year systems costs, a bus driven 50 miles a day costs $20 thousand per year, while one driven twice as far costs only a thousand dollars a year more. In fact, if each bus made more trips, the total mileage travelled would not change much, even though each bus went further.

91-passengers in 3-abreast seating, but for trips over 30 minutes, we assumed that only 2-abreast seating would be used, reducing the capacity to 60.) For the one run we scheduled completely, each bus carried an average of 215 students. The better scheduling techniques resulted in total costs of about $16 million for the central district case.

Resegregation

In the analysis of which student assignments would decrease segregation in the area's schools, we have ignored the problem of resegregation. The results are thus fully valid only if we believe that the ethnic groups will continue to be distributed in the way they were when the data was gathered, and that families will disregard the operations of desegregation in making decisions about whether their children should go to private school or whether they themselves should move. In many communities, parents by their private choice have continued to confound all attempts to achieve meaningful school desegregation*

For this reason, it might be wise to consider alternatives to daily bussing that might meet less community resistance and yet achieve at least some of the advantages of desegregation. One scheme is to establish special

*A major advantage of including the school districts around the central district in the area to be desegregated is that resegregation will be lessened, since it will be more difficult for majority families to move away from minority schools.

campusses for certain specialized subjects such as science, the arts, or physical education, to which students would be transported from a wide area once or twice a week. With specialized facilities and teachers, instruction and exposure in these subjects might be improved while segregation is reduced by student assignment plans.

Another possibility is cooperation and communications between pairs or groups of schools, with pupil exchanges or mass visits on a less than daily basis

Finally, it is important to think of devices that may make forced desegregation more palatable or increase the magnitude of voluntary desegregation. No policy can work well if most people are strongly opposed to it. Large school districts may never overcome the political resistance to mandated bussing; but even if this were to occue at some future time, increased voluntary desegregation in the interim would bring the benefits sooner and ease the transition. Among the devices which have been proposed to encourage voluntary integration are the establishment of very good schools in the inner city areas, provision of free bussing for any family which sends its children to a school where the result is decreased overall segregation and the payment of actual subsidies, if not in cash, then in the form of extra school services or other educational benefits.

Appendix A

MATHEMATICAL ASPECTS OF THE BUSSING PROBLEM

Mathematical Formulation of the Problem

Suppose the area to be designated has been partitioned into N geographical regions. For each region and each level of school (elementary, junior, and high school), we need to know the following:

c_i, the school capacity of the ith region (for that level),

m_i, the number of minority students living in the ith region, and

w_i, the number of majority students living in the ith region.

If grade reorganization is being planned, instead of three levels of schools we must consider 13 (K - 12), since the program obtains desegregation by level rather than by grade.

To apply the maximum travel time constraint, we must know the travel time t_{ij} from each region i to every other region j. Since we consider only the extra time in being bussed as an inconvenience, we let t_{ii}, the travel time for students going to schools in their home regions, be 0. This is equivalent to assuming that, on the average, walks to bus pickup points are as long as walks to schools. Naturally $t_{ij} = t_{ji}$.

Finally, let w_{ij} represent the majority students who live in region i and are assigned to schools in region j, and let m_{ij} represent the minority students living in region i who are assigned to schools in region j.

Obviously, these interregional assignments, which we will call flows, cannot be negative, so we have

(1) $$m_{ij} \geq 0, \; w_{ij} \geq 0.$$

The students who go to schools in their own regions, w_{ii} and m_{ii} are assumed not to be bussed. Every student must go to some school in some region, so that

(2) $$\sum_{j=1}^{N} m_{ij} = m_i \text{ and } \sum_{j=1}^{N} w_{ij} = w_i.$$

We do not permit overcrowding in the schools of any region, so for all j,

(3) $$\sum_{i=1}^{N} m_{ij} + \sum_{i=1}^{N} w_{ij} \leq c_j.$$

There is an upper limit T on allowable bussing times, so that

(4) $$m_{ij} = 0 \text{ and } w_{ij} = 0 \text{ if } t_{ij} > T.$$

The maximal achievable balance within the rules on travel times is given by the solution to the following problem: Find the flows of students, m_{ij} and w_{ij}, which maximize the desegregation index, subject to constraints (1 - 4). The problem is difficult because the objective, DI, is a quadratic function. We use the simple balancing algorithm described below to solve it. The appendix also describes the methods used to prove that the assignments are optimal. For the remaining computations, which develop efficient assignment schedules for lower levels of racial balance, we will set up the problem as a linear program by using the limit proportions approach.

Thus, each region will be required to keep the proportion of minority students at each level of school within certain bounds. These bounds may be the same for every region, but since it is no harder to assume that they are not, we do not require this in the problem formulation. Thus, at each region j, we assume that upper and lower limits on the number of minority students u_j and l_j have been selected. This yields

$$(5) \qquad l_j \sum_{i=1}^{N} (m_{ij} + w_{ij}) \leq \sum_{i=1}^{N} m_{ij} \leq u_j \sum_{i=1}^{N} (m_{ij} + w_{ij}).$$

To avoid infeasible problems, we should choose u_j and l_j to be less restrictive than the solution to the maximal balance problem above. By choosing u_j and l_j equal to their values in the solution of the maximal balance problem, we can improve on the efficiency of that assignment scheme.

Given the constraints (1 - 5), we will try to minimize the total travel time $\sum_{i,j} t_{ij} (m_{ij} + w_{ij})$. This is a linear programming problem and is solved by the IBM "MPS" canned program. To avoid long trips, we can successively multiply the largest t_{ij}'s by a large constant. An alternate scheme, suggested by M. Juncosa, is to add constraints $m_{ij} = 0$ and

$w_{ij} = 0$ successively in order of decreasing t_{ij} until the problem becomes infeasible.

To minimize financial costs with new construction not considered, we will minimize the total number of travelers $\sum_{i,j} (m_{ij} + w_{ij})$, given constraints (1 - 5). Again this is a linear program. The numbers in the solution to this problem could be used as supplies and demands in a standard Hitchcock network problem (where we would then minimize travel times), but this might violate the upper limit on travel times, and so could only be used in somewhat smaller school districts.

A slight change permits us to find the minimal cost plans when new construction is allowed. Let c_j' be the new construction in the jth district. Equation (3) is replaced by

$$(3') \qquad \sum_{i=1}^{N} m_{ij} + \sum_{i=1}^{N} w_{ij} \leq c_j + c_j' .$$

If a represents the average cost of transporting another student and b represents the average cost of increasing capacity by one student, the cost function to be minimized becomes

$$\sum_{i,j} a(m_{ij} + w_{ij}) + bc_j'.$$

This approach assumes that transportation costs and new capacity costs are linear.

The solutions to one problem will help us in starting others as the (simplex) method used makes small adjustments to assignments until an optimum is found. We should expect that the solution for elementary schools will not be proportionately much different than that for high schools and that solutions with slightly different values for T will be similar. Thus, while many different sets of constraints and data will be examined, the computer time used will not grow proportionately to the number of runs.*

Aggregation

To make the problem of scheduling flows tractable, we split the district into regions and assumed temporarily that each region was homogeneous. To check how this assumption distorted the results, we used a more detailed approach. This is important, since it would necessarily be used to complete any real plan.**

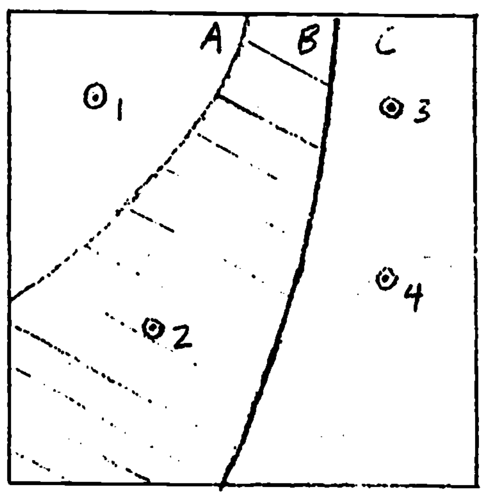
After the flows of students have been assigned, we look at each region in detail. We draw a map, which includes the location of schools, and crude residential data (which we derived from school records). After the flows

*To avoid the problems associated with integer programming, we will assume that children are divisible. Fractional children in the solutions can be rounded up or down. The data cannot be accurate enough for this to matter anyway since children will move in the time between the start and completion of any analysis.

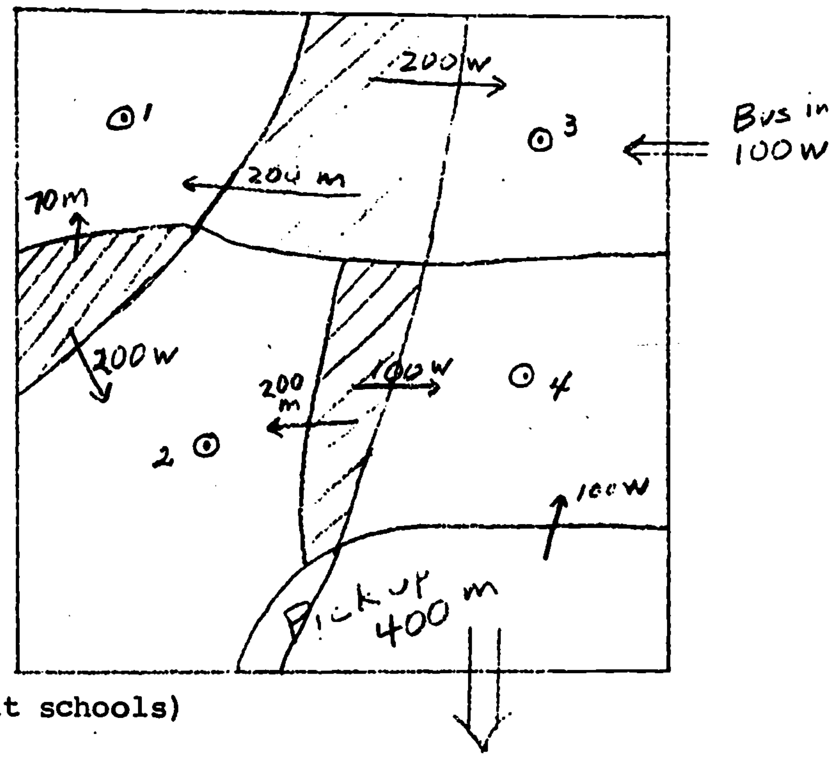
**This tedious procedure should only be used after one desegregation plan has been definitely chosen.

are added to the original numbers of majority and minority students, we can compute the final percent majority for the region. The problem is to get each school in the region to be at that percentage, with as little additional bussing as possible. To do this, we select the pickup points from neighborhoods that have high concentrations of the type of student bussed out. These neighborhoods are also chosen far from the schools so that all remaining students can walk. We select the destination schools to be those that have low concentrations of the type of student bussed in. If imbalances remain, areas between schools are classified as two-way, so that students from the area go to whichever school needs them to improve its ethnic balance. The whole procedure is shown by example in Fig. 4.

Any remaining imbalances can be removed with very few busses, since they can operate as quick shuttles between schools. For the 44-region central city runs, we estimated that only 10 extra busses were necessary to balance all the regions internally.



Map of Region
Before Bussing



(Circled dots represent schools)
W = white
M = minority

Final Boundaries for Internal Balance
(Two-way zones are shaded)

Fig. 4--Example of Balancing Out Schools Within Districts

Data:

Number of Elementary School Students

Initial	Minority	White
Subregion A	200	600
Subregion B	600	600
Subregion C	1200	300
Scheduled Flows		
Out	-400	
In		+100
Final Total	1600	1600

Each school has capacity 800.
Final plan puts 400 white,
400 minority in each school.

Maximum Desegregation*

This section discusses the problem of maximizing desegregation index E, subject to the constraints

(1) $$w_i \geq 0, \quad m_i \geq 0,$$
$$w_i + m_i = c_i$$
$$\sum_{i=1}^{n} w_i = w,$$

and other constraints imposed by the upper limit on allowable student single trip travel times. The capacities, c_i, and the total number of whites, W, are fixed real numbers. We take E to be given by

(2) $$E = \sum_{i=1}^{n} p_i(1 - p_i)c_i, \quad \text{where } p_i = w_i/c_i.$$

We will use an elementary lemma in the arguments.

Lemma. If $p_i < p_j$, E is increased by transferring a sufficiently small number of whites from the j^{th} district to the i^{th} district.

Proof. Since $d/dw_i[w_i(c_i-w_i)/c_i] = -2w_i/c_i = -2p_i$, if x is the number of white students transferred from j to i,

$$\frac{dE}{dx} = -2\,p_i - (-2p_j) = 2(p_j - p_i) > 0.$$

*This section was written by John H. Lindsey II.

The balancing algorithm used to compute Table 1 implements the lemma in the following way. Let us call two regions close if students can be bussed from one to the other within the limit on single trip travel times. The computer compares the percent majority of each region with the percent majority of each region "close" to it. If they differ significantly, a small interchange of majority and minority students is scheduled provided that there remain any original students of the correct sort in each of the two regions. The process is repeated until the solutions converge.*

This may result in a less than optimal allocation of students if the area to be desegregated contains situations like the one illustrated in Fig. 5. To eliminate such sub-optimal allocations, the balancing algorithm next examines, for each region, each pair of close regions it sends students to. If one of the two regions has a significantly higher percent majority, a slight readjustment of the flow is made to bring their percent majority closer together. Thus in Fig. 5, we would assign one fifth more of the first type of student from region B to region C, and a fifth less from region B to region D, which would bring about the best possible allocation. While there exist more intricate situations which can cause even the latter balancing to be less than optimal, such situations did not occur in any of the cases studied.

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*The solutions converge since the desegregation index is bounded above by $N(1-p)p$, and each adjustment increases the index by a positive amount.

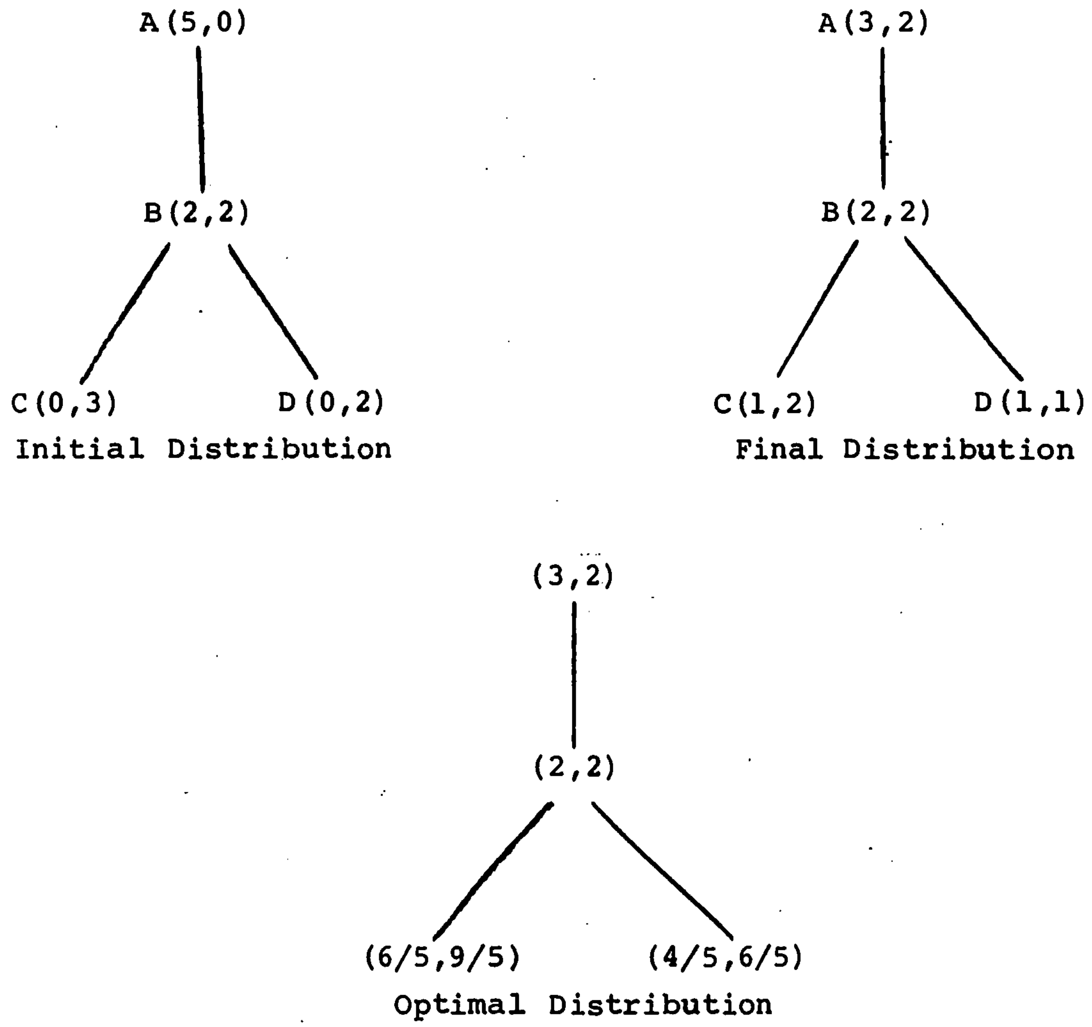


Fig. 5.--Area in Which The Balancing Algorithm Doesn't Work

(The numbers in parenthesis represent the minority and majority students at each location.)

The following theorem usually enables us to prove that the computer found an optimal solution.

THEOREM. Let R^* be a solution $(w_1^*, \ldots, w_n^*, m_1^*, \ldots, m_n^*)$ and let the regions be partitioned into disjoint sets $S_1, \ldots, S_m$ such that $p_i^* \leq p_j^*$ whenever $i \in S_a$, $j \in S_b$, and $a \leq b$. (In particular, $p_i^* = p_j^*$ for i and j in the same S_k.) Let

$$\alpha_k^* = \sum_{i \in S_1 \cup \ldots S_k} w_i^* \quad \text{for } k = 1, \ldots, m.$$

If R' is another solution with $\alpha_k' = \sum_{i \in S_1 \cup \ldots S_k} w_i' \leq \alpha_k^*$ for all k, then $E(R') \leq E(R^*)$.

Proof: Instead of just looking at solutions to the problem, we temporarily do not apply the single trip time limit. That is, we consider all solutions subject only to constraints (1), and $\alpha_k' \leq \alpha_k^*$ for $k = 1, \ldots, m$. The allowable $(w_1', \ldots, w_n', m_1', \ldots, m_n')$ form a compact set in R^{2n}, so there exists a solution $(w_1, \ldots, w_n, m_1, \ldots, m_n)$ maximizing E subject to the above constraints. We shall show that this is the solution R^*, by induction.

For i, j in any S_k, shifting students between i and j will not violate the constraints, so by the lemma, $p_i = p_j$ in the maximum. Suppose, in the maximum that

$$\sum_{i \leq k} w_i < \alpha_k^* \quad \text{for } k < m.$$

Then since small numbers of students can be shifted without violating the constraints, the lemma shows that $p_1 = p_2 \cdots = p_n$. Then, $\alpha_1 =$

$$\alpha_1 = \sum_{i \in S_1} w_i = p_1 \sum_{i \in S_1} c_i = \left(\sum_{i=1}^{n} p_i{}^* c_i^* / \sum_{i=1}^{n} c_i^* \right) \sum_{i \in S_1} c_i .$$

But since the sets S_a, are arranged in order of increasing p_a, the rightmost term is greater than

$$p_1^* \sum_{i \in S_1} c_i^* = \sum_{i \in S_1} w_i^* = \alpha_1^* .$$

This contradicts the assumed strict inequality, so for some $j < m$, we have $\sum_{i \in S_1 \cup \ldots S_j} w_i = \alpha_j = \alpha_j^*$.

We may look at $S_{j+1}, \ldots, S_m$ as a new subcity in itself. Shifting students in this new city will not violate the constraints provided we satisfy (1), and

$$\sum_{i \in S_{j+1} \cup \ldots S_r} w_i \leq \alpha_r - \alpha_j \quad \text{for } r = j + 1, \ldots, m.,$$

By optimality, we cannot increase E by doing so. The part of the solution R* pertaining to the subcity satisfies these constraints, so, by induction on m, is the same as the optimal for the new subcity. The same argument applied to the subcity $S_1, \ldots, S_j$, completes the proof.

The computer's solution so far has always possessed the same properties as were given for R* in the theorem, and hence have been optimal so far. We now give an example of how to prove that any other solution R' satisfies

$$\sum_{i \in S_1 U \ldots U S_k} w_i' \leq \alpha_k = \sum_{i \in S_1 U \ldots U S_k} w_k$$

We consider the 44 elementary school regions with 45 minutes being the maximal allowed travel time.

Here $S_1 = \{43\}$ $p_i = 0$ (No schools in this region)

$S_2 = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,$
$21,22,23,25,26,27,29,33,34,35,39,44\}$ $p_i = 44.7.$

$S_3 = \{24,\}$ $p_i = 51.7.$

$S_4 = \{28\}$ $p_i = 64.4.$

$S_5 = \{30,31,32,36,37,38,40,41,42\}$ $p_i = 66.7.$

We show as an example from the matrix in the output, reproduced as Fig. 6, that

$$\sum_{i \in S_1 U S_2} w_i' \leq \sum_{i \in S_1 U S_2} w_i$$

where $(w_1, \ldots, w_{44})$ is the computer solution and $(w_1', \ldots, w_{44}')$ is any other solution. We do this by showing that $S_1 \cup S_2$ sends as many blacks as possible to $S_3 \cup S_4 \cup S_5$.

Regions in $S_1 \cup S_2$ Within Range of $S_3 \cup S_4 \cup S_5$	Original Blacks in Region	Blacks Not Sent to $S_3US_4US_5$	24	28	30	31	32	36	37	38	40	41	42
Regions in $S_3US_4US_5$ in Range of S_1US_2			24	28	30	31	32	36	37	38	40	41	42
Original Whites in Region			2968	1872	12499	11852	11620	3278	8133	8859	4311	5492	5842
Whites Not Sent to $S_1 \cup S_2$			0	0	11348	10918	10160	2751	6728	8295	4268	4486	5297
5	7969	7655	192	121									
6	10778	10310	281	186									
8	9652	9232	251	168									
9	15400	14890	310	199									
10	2054	1996	58										
12	8816	8459	217	139									
13	13235	12743	296	195									
14	3199	3072	81	45									
15	1113	0	0			316	275		152	35			332
16	7127	6999	84	42									
17	9637	9295	207	135									
18	5411	5411	0	0									
19	10083	9500	358	224									
20	7300	6892	284	185									
21	62	0					38		10	10			2
22	2118	0	0	0			468		537	247		749	64
23	794	0	0	0			105		143	213		240	91
25	8053	7567	298	187									
26	128	0	0	0	112	3	1	8	0	0		0	0
27	306	0	0	0	234	43	21		2	1	1	0	0
29	2189	2189	0	0									
33	487	0	0	0	185	63	70	86	76	0	4	0	0
34	835	0	0	0	321	105	112	170	117	1	5	0	0
35	931	0	0	0	296	158	166	126	176	0	5	0	0
39	843	0	0	0		242	198	134	188	2	26	15	35
43	0	0							0	0	0	0	0
44	8894	8810	44	39									

Fig. 6.--Computer Output Matrix Used to Prove Optimality
(Entries in main body of matrix give exchanges between schools.)

The top row contains the regions in $S_3 \cup S_4 \cup S_5$ which are within allowed travel time for some regions in $S_1 \cup S_2$. The second column lists the regions in $S_1 \cup S_2$ which are within allowed travel time of some district in $S_3 \cup S_4 \cup S_5$. The i^{th} row and j^{th} column of the main part of the matrix gives the number of blacks, the region in the i^{th} row sends to the region of the j^{th} column (and hence, the number of whites it gets in return). By looking at the row of whites not sent to $S_1 \cup S_2$, we see that regions 24 and 28 send all their whites to $S_1 \cup S_2$. These whites go to T = {5,6,8,9,10,12,13, 14,16,17,18,19,20, 25,29, and 43} since the other rows all have 0's opposite 24 and 28. But outside this set T, the 0's in the column "blacks not sent to $S_3 \cup S_4 \cup S_5$" show that regions in $S_1 \cup S_2$ send all their blacks to $S_3 \cup S_4 \cup S_5$. However, regions in T connect only to $S_3 \cup S_4 \cup S_5$. This is shown in the output by blanks. (If schools were connected, there would be 0's printed.) The regions in T saturate the capacity of 24 and 28 to absorb blacks, and so T sends as many blacks to $S_3 \cup S_4 \cup S_5$ as possible. Therefore, the computer's solution $\Sigma\, w_i$ is maximal given the travel time constraint.

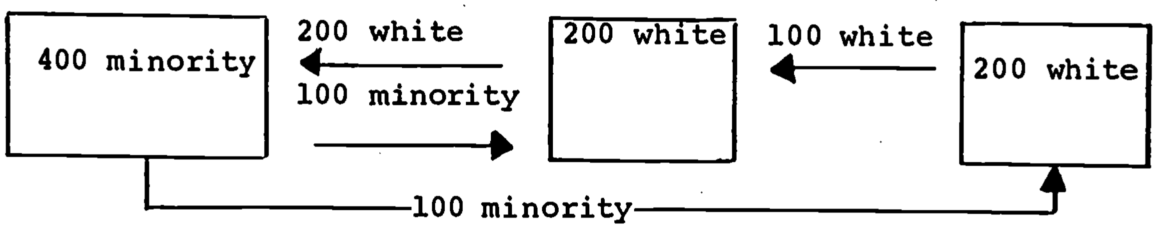
Maximum Bussing Required in Small Districts

If any student could be bussed to any school, then every school could have the percent white that the whole area had. Such would be the case if there were no limit on individual travel times, or if the region were small enough so the limit did not rule out any trip.

If P represents the percent white of the whole area, the i^{th} school is out of proportion by $(w_i + m_i)\ |P - p_i|$ students. Thus the total needed to be bussed is $\sum_i (w_i + m_i)\ |P - p_i|$. With total segregation (each p_i is 0 or 1), the number bussed is 2NP(1-P), which has a maximum of 50%, when P is 50%.* Intuitively, more than half must stand still, and the rest must be bussed to them.

Example of Optimal Flows Which are not Paired

Bus scheduling is easier when flows are paired so that for each white sent from one school to another, there is a minority student coming back. However, even when there is no overcrowding, optimal flows may not be paired in this way. In the example below, boxes represent student residences, and arrows represent flows which achieve total desegregation while minimizing total travel time. The flows are clearly not paired.



*With n groups to be desegregated, at most (n-1)/n of the students must be bussed.

Appendix B

BUS SCHEDULING

Even after we have generated an efficient set of flows which reduce segregation the desired amount, there is still the problem of assigning busses to carry these flows. Bus scheduling between regions is the critical step in keeping down costs. Within each region, we select for bussing neighborhoods far from schools, to permit as much walking to school as possible and thus reduce additional bussing.

Staggered Starting Times

If all schools start and finish at the same time, busses are used very intensively at those times, and are idle the rest of the day. Since drivers are normally paid a full day's pay and the purchase price and parking represent most of the busses' cost, it saves money to use the busses more of the day. This can be done by staggering school hours. The staggering can be done by level of school, by geographical area and within individual schools.

Perhaps the easiest to administer would be staggering by level of school. Table 6 shows three plans. We make the approximation that total bussing is divided 60%, 20%, 20% between Elementary, Junior High, and High School students. The last column gives the busses needed at that time as a percentage of the busses needed to transport everyone at

Table 6

BUSSING ECONOMIES BY SCHOOL HOUR STAGGERING PLANS

Grades	Starting Time	Finishing Time	Busses Needed as % of Total
		Plan 1	
1-3	7:30	12:30[a]	30[b]
10-12	8:15	2:15	20
4-6	9:00	3:00	30
7-9	9:45	3:45	20
		Plan 2	
1-2, 10-12	7:40	12:45,[a] 1:45	40[b]
3-6	8:40	2:40	40
7-9	9:40	3:40	20
		Plan 3	
7-9, 10-12	8:00	2:00	40
1-2, 3-6	9:00	2:00,[a] 3:00	60[b]

[a]Time based on the assumption that children in the early grades only go to school 5 hours a day.

[b]Maximum percentage of all at once bussing required by plan.

once. The plans all have the advantage that Senior High School students can stay one and a half hours after school, and then be bussed home without raising the maximum percentage of busses needed.

In addition to staggering by level, it may be useful to stagger by area. For example, in a city like the one in Table 7, if we start each level one trip length later in the suburbs than in the center city, the busses can be in continual use. In this case, we could use one sixth of the number of busses needed to transport all the students at once.

One problem with this method is that irregularities in the needed flows will occur, if flows in and out of a region are not equal [some schools might be currently overcrowded, and some undercrowded, because of shifting population in in the area.]

A final possibility is to have different students come at different times to the same school. By double sessions, or partial double sessions, class size can be decreased for subjects in which this facilitates teaching. This will permit increased trips per bus and reduce the number of busses necessary. The simplest flows of students to schedule busses for are paired flows. If the same number of students are switched between regions, one can simply have busses going forwards and backwards on the same routes.

Table 7

AN EXTREME CASE OF MULTIPLE BUS USE

(Grades 1-3 are on 3 hour session.
Everyone else on 5 hour session.)

Predominantly Minority Part of Town	Travel Time (Including loading and unloading = ½ hour)	Predominantly Majority Part of Town
1–3 8:00–11:00 4–6 11:00– 4:00	Elementary Schools	1–3 7:30–10:30 4–6 10:30– 3:30
9:00–2:00	Junior High Schools	8:30–1:30
10:00–3:00	Senior High Schools	9:30–3:30

Each bus is used for eight trips afternoon and evening. The bus is driven from 7:00–11:30 and 1:30–5:00.

However, such two-way flows were not feasible for our computer runs for two reasons. First, the number of students living in a region was obtained from the census, and the school capacities from 1969 school enrollments. The overcrowding was assumed to be spread evenly throughout the system. Thus, to ensure that each school had the right number of students required some one-way bussing. In addition, as is shown in Appendix A, it is not possible to achieve as good a balance with two-way flows as with all flows, and two-way flows may be wasteful in terms of number of students [not numbers of busses] moved. In the next section we describe a computer algorithm for scheduling one-way flows. This algorithm is considerably more effective than doing the scheduling by hand, but has three major defects:

1) Since it uses linear programming, and not integer programming, the number of busses scheduled on any route may be fractional, and hence must be rounded up to the next integer. Thus the solution, while good, is not optimal.

2) If the period chosen for scheduling busses is too long, the number of possible trips becomes too large to deal with economically. Thus, rather than trying to schedule Elementary, Junior High School, and Senior High School all at once, it is better to stagger starts as in Plan 2, and use a 45-minute limit. This increases bus requirements somewhat, but greatly reduces the administrative complexities.

3) If a two-link trip does not double back, then the afternoon trip will go backwards along the same links. The people picked up last in the morning, get home first at night. If this is sufficiently undesirable, it may be best to constrain the flows to two-way pairs, or just not use such two-link trips. In a sample case, with a 45-minute time limit, we estimated that 1900 links could be covered with 1400 busses in 50 minutes using any allowable trips, but required 1700 busses if only two double trips were permitted.

From the point of view of cost and administrative ease, the best method is probably to compute the flows using only two-way equal but opposite flows. This would mean that overcrowding should be ironed out before the main computation starts. The schools could use the easily obtainable school enrollment figures as the starting point.

In general, the number of busses can be reduced by altering school schedules and increasing the number of students bussed, but past a certain point the complexities that optimization requires aren't worth it.

Formulation of Bus Assignment as a Linear Program

Given a set of student flows, we want to develop a bus assignment plan that minimizes the number of busses needed. The plan must satisfy constraints on school schedules and how long children may wait in school yards.

To make the problem manageable, we make the simplifying assumption that fractional busses are allowed. Small fractions can be ruled out by the (computationally simple) requirement that the smallest assignment to a route is one bus. The student flow assignments come from a linear program, so that there are only about one hundred flows to consider, but there will be a slight loss of efficiency due to the rounding up of nonintegral bus assignments. This efficiency loss is almost inevitable. Except for certain very special classes of problems, it is usually difficult and costly to solve integer programming problems exactly.

Let $\{n_{ij}\}$ be the required set of student flows between the N regions of the area. The scalar n_{ij} represents the total number of students who have been scheduled to go from region i to region j.

The only bus trips that need be considered are those in which a bus loads completely at the starting region of the first link, unloads completely at the finishing region of the first link, drives empty (deadheads) to the starting region of the second link (if it is different from the finishing region for the first link).

Definitions and notation. A link is an ordered pair of regions such that students' flows have been scheduled from the first region to the second. A bus trip can be defined simply as a set of links $(\ell_1, \ell_2, \ldots, \ell_k)$. A bus is said to make this trip if it loads completely at the starting region for ℓ_1, unloads completely at the finishing region of ℓ_1, deadheads to the starting region of ℓ_2 (if it is different from the finishing region for ℓ_1), loads completely and so forth. With fractional busses allowed, this is the only type of trip that need be considered, since partial loading trips can be transformed into complete loading and unloading trips with fractional busses. For example, if a bus loads completely at region 1, and unloads half at region 2 and half at region 3, we may consider it to be two busses of size 1/2, one unloading at region 2 and one at region 3.

The trips t_j under consideration give rise to an incidence matrix A. In the example matrix of Fig. 7, t_2 represents the bus which loads completely at region 1 (the starting region of ℓ_2) and unloads completely at region 3 (the finishing region for ℓ_2), and t_4 represents a bus which loads completely at 1, unloads at 2, deadheads to 1, reloads and then unloads at 2.

Let $\{n_k\}$ be the required flows between the N regions of the area. That is, n_k represents the number of students who must go on link k. Let x_j be the number of busses assigned to trip t_j.

<u>TRIP INCIDENCE MATRIX</u>

Link	Starting Region	Finishing Region	t_1	t_2	t_3	t_4	t_5
ℓ_1	1	2	1	0	0	2	1
ℓ_2	1	3	0	1	0	0	0
ℓ_3	2	3	0	0	1	0	1

A

Fig. 7

The linear programming problem can now be stated. Find $x_i \geq 0$, to minimize $\sum_i x_i$ such that

$$(1) \qquad (x_1, \ldots, x_k) \cdot A \geq \begin{pmatrix} n_1 \\ \vdots \\ n_\ell \end{pmatrix}.$$

Normally, there will be an upper limit on allowable trip times, or a penalty assigned to over-long trips. The trip limit is in addition to the upper limit on student travel times (a link limit). The trip limit comes from requirements on how long students may wait at schools before they start and from the staggered school starting hours which forces the bus schedules for different levels of schools to fit together.

Let $t(\ell_i)$ be the time required to travel link ℓ_i (including loading and unloading) and $t_d(ij)$ be the deadhead time from the finish of link i to the start of link j. A trip $(\ell_1, \ell_2, \ldots)$ is allowable if

$$(2) \qquad \Sigma \; [t(\ell_i) + t_d \; (ij)] \leq T_{max}$$

When this formula characterizes the allowable trips, the sum of deadhead times

$$(3) \qquad \Sigma \; x_i \; (\Sigma \; t_d \; (jk))$$

can replace $\Sigma \; x_i$ as the function to be minimized. The solution to both problems is the same, but the deadhead times formulation has better convergence properties.

If the time limit, T_{max}, is not too large,* a computer can easily generate all possible trips subject to (2), and to a limit on individual deadhead times, given by intuition. These trips can be used as variables in the linear program (1), or that program with deadhead times (3) replacing $\Sigma \; x_i$ as the objective function.

However, if trips can contain many links, there will be too many for this approach to be practical. In that case, we can guess which trips may be good and use them to solve the problem.**

We can then use the prices from the dual problem to guess better trips which we will enter as variables in the next attempt at solution. (Although there are systematic ways to find a best path quickly, to make progress we should get twenty or thirty of the best. Thus intelligent guessing is probably best.)

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*So that few trips can have more than three links.

**The set of trips which repeat each needed link as often as possible by deadheading from the start is feasible and hence may be a good first iteration choice.

Appendix C*

BUS COSTS

General

Bus costs were broken down into: investment (or capital) costs for all land, buildings, busses, and other equipment required for the operation of the bus fleet; and annual operating costs, consisting of salaries of all personnel and all other expenses.

Bussing costs were developed from data for a representative city school district. Costs for busses, land costs for parking areas, salary scales for drivers and administrative personnel, and bus fuel and maintenance costs came from a 1971 report. Supplementary factors and cost relationships come from an earlier report for the same district, written in December, 1968.

Investment Costs

Investment costs include: busses; parking facilities; maintenance facilities (including shop equipment); other vehicles (including care and trucks) and office equipment.

Bus unit costs, costs per acre for parking, and the number of busses that can be parked per acre came from the 1971 report. Factors for maintenance facilities costs and

*This appendix was written by Annette Bonner.

other vehicle requirements were developed from the 1968 study and added to the bus and parking costs. A 10 percent allowance was made for spare buses (as in the 1968 report).

We considered just a 91-passenger diesel-powered bus, rather than smaller buses, in order to reduce the number of buses required. This bus holds 91 passengers in 3-abreast seating; but for longer trips 2-abreast seating is desirable, reducing capacity to 60 passengers. The unit cost of these buses is $45,140.

An acre of land is required to park 40 buses of this size. Land costs are approximately $87,500 per acre in the suburban areas and $108,900 in the inner city. We assumed half of the buses would be parked in the suburbs and half in the inner city, so the average cost per acre is $98,200.

Maintenance facilities, including shop equipment, buildings, land, etc. were factored from the 1968 report on the basis of one central overhaul garage and 6 satellite garages for an estimated 1,660 buses. The factor developed is 1.5 times the cost of land requirements for parking.

Other motor vehicles and office equipment were factored as a percent of bus costs (from the 1968 study). This factor is .005 times the cost of the buses.

A 10 percent allowance for spare buses was added to this total.

The resulting costs per operational bus are:

	Factor	Cost per Bus
Bus (91-passenger)		$45,140
Parking	98,200/40	2,455
Maintenance Facilities	1.5(2,455)	3,682
Other Vehicles and Office Equipment	.005(45,140)	226
Total		$51,503
Allowance for Spare Buses	.10(51,503)	5,150
Total Cost per Operational Bus		$56,653

Annual Operating Costs

Salaries

The salaries include salaries and all fringe benefits for drivers, and all administrative, support, and maintenance personnel required to operate and support the bus fleet.

The 1971 report provided hourly wage schedules by type of personnel for varying lengths of service, but there were no distributions of personnel in each of the categories. Nor was there any information as to the number of support and administrative personnel necessary for a given number of bus drivers. We developed some of these factors from the 1968 report, and estimated the rest.

We estimated the average length of services of bus drivers to be 2 years (the hourly rate for that length of service is $4.71). We assumed that drivers were paid for 8 hours per day for 200 days (the length of the school year including holidays). From the 1968 report we found that there were 6 percent more drivers than operational buses.

All other salaries plus fringe benefits for drivers and all other personnel were derived as a factor of drivers salaries on the basis of the data in the 1968 report. This factor equaled .6 times the drivers salaries.

The resulting annual costs per driver are:

	Factor	Cost per Driver
Drivers' Salaries	$4.71(8)200	$7,536
Salaries of Other Personnel Plus Total Fringe Benefits	.6(7,536)	4,522
Total annual Salaries per Driver		$12,058
Annual Cost per Operational Bus	1.06(12,058)	$12,781

Other Expenses

This includes operation and maintenance of buses and other vehicles; other maintenance related to the operation of the bus fleet; insurance on buildings, vehicles, and equipment; and garage overhead (excluding salaries).

Maintenance and operation costs per mile came from the 1971 report (based on 1969-70 data, the latest available) for 91-passenger buses. These costs totaled $.125 per mile.

Costs of maintenance and operation of other vehicles were factored from the 1968 report. These amounted to 1.2 percent of the annual cost of maintenance and operation of the buses.

Insurance, garage overhead, office supplies, etc., are more properly related to the number of buses, rather than to the maintenance and operation costs of the buses. Costs for insurance, etc., came from the 1968 report. The average annual cost per operational bus (based on 1,505 buses) is $1,394,484/1,505 = $927 per bus per year. To this we added an estimated 5 percent allowance for price increases. This was done here and not elsewhere because all other factored costs are related to updated unit costs (e.g., bus unit costs; current salaries, etc.) and presumably reflect the price increases from 1968 to 1971.

The resulting total Other Expenses are:

	Factor	Cost per Mile	Annual Cost per Bus
Bus Maintenance and operation		$.125	
Other Vehicle Maintenance and Operation	.012(.125)	.002	
Total Vehicle Maintenance and Operation		$.127	
Insurance, Garage Overhead, Office Supplies, etc.	1.05(927)		$973

Total Investment and Annual Operating Costs

	Cost per Bus	Cost per Mile
Investment	$56,653	
Annual Operating Cost		
Salaries	12,781	
Other Expenses	973	$.127
Total Annual Operating Cost	$13,754	$.127

Depreciation

Three methods of handling depreciation are considered. The usual Rand costing methodology presents a 10-year systems cost (investment cost plus 10 years operating cost). This assumes that all capital equipment is amortized in 10 years with no allowance for salvage or trade-in.

The 1968 report used a 5-year systems cost (5-year amortization), and also presented one case where depreciation was computed for each type of building, piece of equipment, and vehicle.

Since a 10-year amortization may not be appropriate in this case, we have also computed an average cost per bus assuming a 5-year amortization of capital equipment, and we have estimated actual depreciation, using the same methodology and factors developed from the 1968 report, but using the 1971 bus costs.

In computing the actual depreciation on capital equipment, bus depreciation was figured on the basis of a 25-year life and a trade-in value of $1,000. This was then inflated by the 10 percent allowance for spare buses, in order to express depreciation in terms of operational buses. A factor reflecting depreciation of all other capital equipment as a percent of bus depreciation was developed from the costs in the 1968 report: it is 289,751/2,629,808 = 11.0 percent.

The resulting total depreciation per operational bus is as follows:

	Factor	Cost per Bus
Bus Depreciation	$\frac{1.1(45,140-1,000)}{25}$	$1,942
All Other Capital Equipment	.11(1,942)	214
Total Depreciation		$2,156

Average Annual Costs Per Bus

Five-Year Amortization of Capital

The average annual cost per bus is the investment cost divided by 5 plus 1 year's operating cost. These costs are expressed as a cost per operational bus, with the daily mileage per bus kept as a variable "m." Although the school extends for 200 days, school is in session only 179 days.

The average annual cost per bus, A, (in dollars) based on 5-year amortization is:

$$A = \frac{56,653}{5} + 13,754 + 179(.127)m$$

$$A = 25,085 + 22.73m$$

Ten-Year Amortization of Capital

The average annual cost per bus in dollars when capital is amortized over 10 years is:

$$A = \frac{56,653}{10} + 13,754 + 22.73m$$

$$A = 19,419 + 22.73m$$

Allowance for Computed Depreciation

In this case the average annual cost per bus is the annual depreciation plus 1 year's operating cost in dollars.

$$A = 2{,}156 + 13{,}754 + 22.73m$$

$$A = 15{,}910 + 22.73m$$

Total Costs Per Bus Per Year

For all the plans considered, the buses were scheduled to drive between 50 and 100 miles per day (including deadhead). If buses are scheduled to make many trips, each bus travels farther, but not so many are needed. The overall distance traveled remains about the same.

The following table shows the range in costs per bus for each of the three amortization policies when the daily mileage per bus is 50 and 100 miles.

Amortization Policy	Daily Mileage per Bus 50	100
5-Year	$26,222	$27,358
10-Year	$20,556	$21,692
Actual Depreciation	$17,047	$18,183

Although the daily mileage per bus doubles, the average total costs per bus only increase from 4 to 7 percent. The average total costs per bus are much more sensitive to amortization policy; costs based on the 5-year amortization are from 50-54 percent greater than when depreciation is computed.

Table 8 shows representative fleet costs to make trips totaling 100,000 miles (including deadhead), using 10-year amortization. The costs are almost proportional to the number of buses, pointing out the importance of efficient bus scheduling.

Table 8

Representative Fleet Costs
to Make Trips Totaling 100,000 Miles

Number of Buses	Daily Mileage per Bus	Cost per Bus (Dollars)	Total Fleet Cost (Dollars)
2,000	50	20,556	41,112,000
1,750	57	20,715	36,251,250
1,500	67	20,942	31,413,000
1,250	80	21,237	26,546,250
1,000	100	21,692	21,692,000